Cambright Solved Paper

Tags	2023 Additional Math CIE IGCSE May/June P1 V3
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Status	Done

1 (a) Write down the period, in radians, of $3\tan\frac{\theta}{2} - 3$. [1]

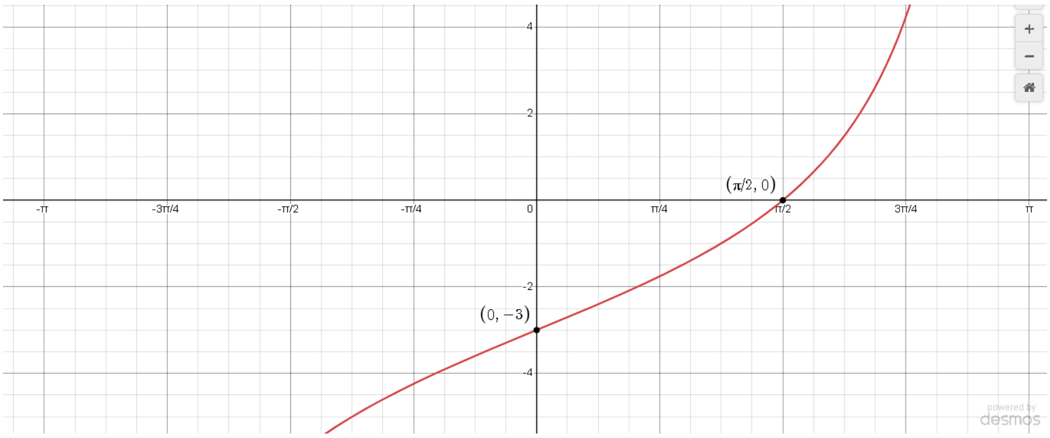
$$period\ of\ \tan\ graph = \frac{\pi}{b}$$

$$period = \frac{\pi}{\frac{1}{2}} = 2\pi$$

(b) On the axes, sketch the graph of $y = 3\tan\frac{\theta}{2} - 3$ for $-\pi \leqslant \theta \leqslant \pi$, stating the coordinates of the points where the graph meets the axes. [3]

When $\theta = 0$, $y = -3$ (y-intercept)

When $y = 0,\ \theta = \frac{\pi}{2}$ (θ-intercept)



https://www.desmos.com/calculator/5jzzhf0rjr

Make sure to label the intercept points!!

2 **(a)** Write $2x^2+5x+3$ in the form $2(x+a)^2+b$, where a and b are rational numbers. [2]

$2(x^2+\frac{5}{2})+3$

$=2[(x+\frac{5}{4})^2-(\frac{5}{4})^2]+3$

$=2(x+\frac{5}{4})^2-2(\frac{25}{16})+3$

$=2(x+\frac{5}{4})^2-\frac{25}{8}+3$

$=2(x+\frac{5}{4})^2-\frac{1}{8}$

(b) Hence write down the coordinates of the stationary point on the curve $y=2x^2+5x+3$. [2]

In complete square form (i.e $2(x+\frac{5}{4})^2-\frac{1}{8}$), when the bracket is equal to 0, we get our x-coordinate for the stationary point.

$x=-\frac{5}{4}\rightarrow y=-\frac{1}{8}$

$Stationary\ point\ =(-\frac{5}{4},-\frac{1}{8})$

(c) Solve the inequality $2x^2+5x+3<\frac{15}{8}$. [3]

Using part a,

$2(x+\frac{5}{4})^2-\frac{1}{8}<\frac{15}{8}$

$2(x+\frac{5}{4})^2<2$

$(x+\frac{5}{4})^2<1$

$x+\frac{5}{4}<\pm1$

$x<\pm1-\frac{5}{4}$

$-1-\frac{5}{4}<x<1-\frac{5}{4}$

$-\frac{9}{4}<x<-\frac{1}{4}$

3 **(a)** Write $3+2\lg a-\frac{1}{2}\lg(4b^2)$, where a and b are both positive, as a single logarithm to base 10. Give your answer in its simplest form. [3]

Using log rules, we can change $\frac{1}{2}\lg(4b^2)$ into $\lg(4b^2)^{\frac{1}{2}}$ which can be simplified to $\lg(2b)$

$3+2\lg a-\lg(2b)$

$3=3\lg 10=\lg 10^3=\lg 1000$ and $2\lg a=\lg a^2$

$\lg 1000+\lg a^2-\lg 2b$

Using the log rules, we can combine these logs into one

$\lg \dfrac{1000a^2}{2b}$ leading to $\lg \dfrac{500a^2}{b}$

(b) Given that $2\log_c 3 = 7 + 4\log_3 c$, find the possible values of the positive constant c, giving your answers in exact form. [5]

Using change of base rule, we can change $\log_c 3$ into $\dfrac{1}{\log_3 c}$

$2 \times \dfrac{1}{\log_3 c} = 7 + 4\log_3 c$

Multiply both sides by $\log_3 c$

$2 = 7\log_3 c + 4(\log_3 c)^2$

$Let\ x = \log_3 c$

$2 = 7x + 4x^2$

$4x^2 + 7x - 2 = 0$

$(4x - 1)(x + 2) = 0$

$x = \dfrac{1}{4}\ or\ x = -2$

$\log_3 c = \dfrac{1}{4}\ or\ \log_3 c = -2$

$c = (3)^{\frac{1}{4}}\ or\ c = 3^{-2}$

$c = 3^{\frac{1}{4}}\ or\ c = \dfrac{1}{9}$

4 The straight line $y = 3x - 11$ and the curve $xy = 4 - 3x - 2x^2$ intersect at the points A and B. The point C, with coordinates $(a, -8)$ where a is a constant, lies on the perpendicular bisector of the line AB. Find the value of a. [8]

Substitute y from line equation to curve equation

$x(3x - 11) = 4 - 3x - 2x^2$

$3x^2 - 11x = 4 - 3x - 2x^2$

$5x^2 - 8x - 4 = 0$

$(x - 2)(5x + 2) = 0$

$x = 2\ or\ x = -\dfrac{2}{5}$

Sub these values in $y = 3x - 11$ to find the y-coordinates

$x = 2 \rightarrow y = 3(2) - 11 = -5$

$x = -\dfrac{2}{5} \rightarrow y = 3(-\dfrac{2}{5}) - 11 = -\dfrac{61}{5}$

Now, we have points A and B, $(2, \ -5)$ and $(-\frac{2}{5}, \ -\frac{61}{5})$

$$\text{Midpoint AB} = (\frac{2-\frac{2}{5}}{2}, \ \frac{-5-\frac{61}{5}}{2}) = (\frac{4}{5}, \ -\frac{43}{5})$$

$$\text{Gradient AB} = \frac{rise}{run} = \frac{-\frac{61}{5}+5}{-\frac{2}{5}-2} = \frac{-\frac{36}{5}}{-\frac{12}{5}} = 3$$

In perpendicular line, $m_1 m_2 = -1 \rightarrow 3m_2 = -1 \rightarrow m_2 = -\frac{1}{3}$

To find the perpendicular bisector's equation, we can use the point slope form

$$y - (-\frac{43}{5}) = -\frac{1}{3}(x - \frac{4}{5})$$
$$y + \frac{43}{5} = -\frac{1}{3}(x - \frac{4}{5})$$

When $y = -8, \ x = a$

$$-8 + \frac{43}{5} = -\frac{1}{3}(x - \frac{4}{5})$$
$$\frac{3}{5} = -\frac{1}{3}(x - \frac{4}{5})$$
$$-\frac{9}{5} = x - \frac{4}{5}$$
$$-\frac{9}{5} + \frac{4}{5} = x$$
$$x = -1$$

Therefore a = -1

5 **(a)** Find the first three terms in the expansion of $\left(x^2 - \frac{4}{x^2}\right)^{10}$ in descending powers of x. Give each term in its simplest form. [3]

$$(x^2)^{10} + {}^{10}C_1(x^2)^9(-\frac{4}{x^2}) + {}^{10}C_2(x^2)^8(-\frac{4}{x^2})^2 + ...$$
$$(x^{20}) + 10(x^{18})(-\frac{4}{x^2}) + 45(x^{16})(\frac{16}{x^4}) + ...$$
$$x^{20} - 40x^{16} + 720x^{12} + ...$$

(b) Hence find the coefficient of x^{16} in the expansion of $\left(x^2 - \frac{4}{x^2}\right)^{10}\left(x^2 + \frac{2}{x^2}\right)^2$. [3]

Using part a,

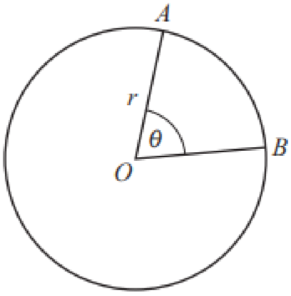
$$(x^{20} - 40x^{16} + 720x^{12} + ...)(x^4 + 4 + \frac{4}{x^4})$$
$$x^{24} + 4x^{20} + 4x^{16} - 40x^{20} - 160x^{16} - 160x^{12} + 720x^{16} + ...$$

Collect terms

$4x^{16} - 160x^{16} + 720x^{16} = 564x^{16}$

Coefficient of $x^{16} = 564$

6 In this question lengths are in centimetres and angles are in radians.



The diagram shows a circle with centre O and radius r. The points A and B lie on the circumference of the circle. The area of the minor sector OAB is $25\,\text{cm}^2$. The angle AOB is θ.

(a) Find an expression for the perimeter, P, of the minor sector AOB, in terms of r. [3]

$\text{Perimeter of minor sector} = r + r + \text{Arc length} = 2r + r\theta$

Since we want to find the perimeter in terms of r, we have to find an expression for θ to use for the arc length.

$\text{Area} = \frac{1}{2}r^2\theta$

$25 = \frac{1}{2}r^2\theta$

$r^2\theta = 50$

$\theta = \frac{50}{r^2}$

$\text{Perimeter of minor sector} = 2r + r\theta$

$\text{Perimeter of minor sector} = 2r + r\frac{50}{r^2}$

$\text{Perimeter of minor sector} = 2r + \frac{50}{r}$

(b) Given that r can vary, show that P has a minimum value and find this minimum value. [4]

$P(\text{Perimeter}) = 2r + \frac{50}{r}$

$\frac{dP}{dr} = 2 - \frac{50}{r^2}$

Minimum value will occur when $\frac{dP}{dr} = 0$

$2 - \frac{50}{r^2} = 0$

$2 = \frac{50}{r^2}$

$r^2 = 25$

$r = 5$

$$\frac{d^2P}{dr^2} = \frac{100}{r^3}$$

Using the second derivative test

$$r = 5 \rightarrow \frac{d^2P}{dr^2} = \frac{100}{5^3} = \frac{4}{5}$$

Since the second derivative is positive, r = 5 is a minimum value.

Now, we can find P when r = 5

$$r = 5 \rightarrow P = 2(5) + \frac{50}{5}$$

$$P = 10 + 10 = 20$$

7 The table shows values of the variables x and y which are related by an equation of the form $y = Ax^b$, where A and b are constants.

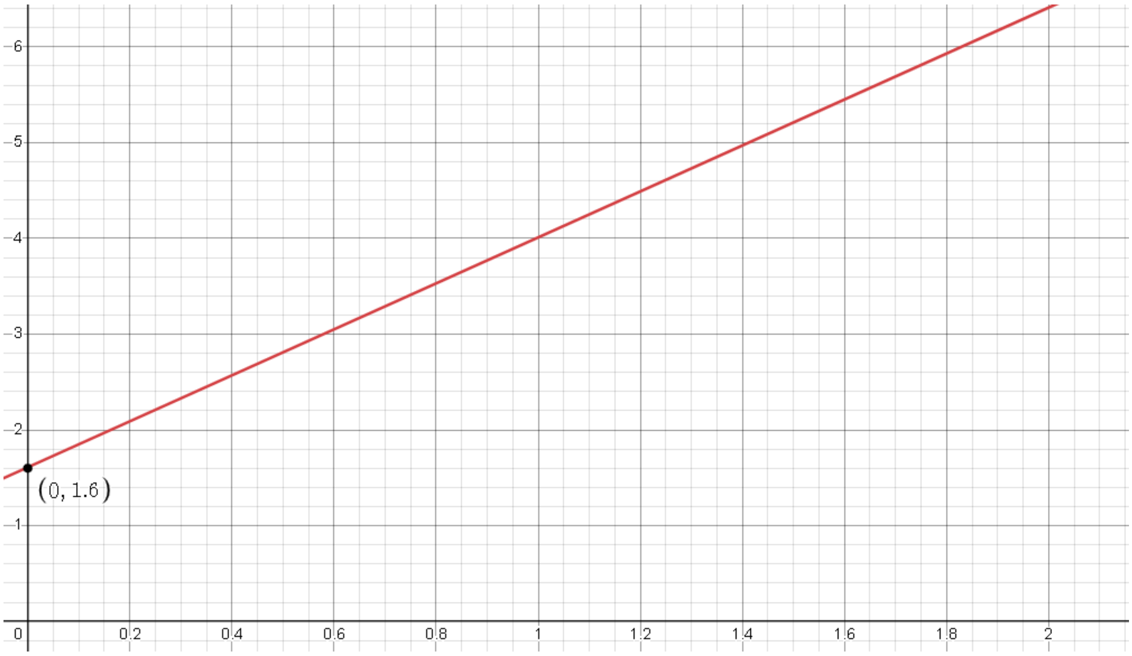
x	1.5	2	2.5	3	4
y	13.8	27.5	46.9	72.6	145

(a) Use the data to draw a straight line graph of $\ln y$ against $\ln x$. [3]

First make a table of values for ln y and ln x

$\ln x$	0.405	0.693	0.916	1.099	1.386
$\ln y$	2.625	3.314	3.848	4.285	4.977

Now, plot the points and draw a straight line



https://www.desmos.com/calculator/jkbactzzof (Note, in desmos, I had to draw a graph with x and y instead of lnx and lny but the thought process remains the same)

(b) Use your graph to estimate the values of A and b. [5]

$y = Ax^b$ is given

Taking the natural log of both sides,

$\ln y = \ln(A \times x^b)$

Using log rules, we can separate the right hand side

$\ln y = \ln A + \ln x^b$

$\ln y = b \ln x + \ln A$

Now, we have a straight line equation form and we can use our graph

$\text{Gradient } b = \frac{4.977 - 2.625}{1.386 - 0.405} = 2.398... = 2.4$

$y - intercept = \ln A$

On your graph, you should see that the y-intercept is around 1.5 to 1.7

We will take y-intercept = 1.6

$\ln A = 1.6$

$A = e^{1.6}$

$A = 4.95... = 5$

(c) Estimate the value of x when $y = 100$. [2]

$y = 5x^{2.4}$

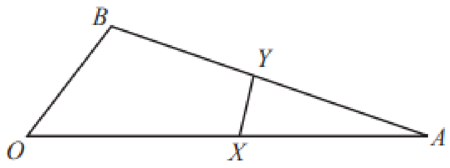
$y = 100 \rightarrow 100 = 5x^{2.4}$

$x^{2.4} = 20$

$x = \log_{2.4} 20$

$x = 3.42$

8



The diagram shows the triangle OAB with $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. The point X lies on the line OA such that $\overrightarrow{OX} = \frac{3}{5}\mathbf{a}$. The point Y is the mid-point of the line AB. Find, in terms of **a** and **b**,

(a) $\overrightarrow{AB}$ [1]

(b) $\overrightarrow{XY}$. [2]

(a) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = b - a$

(b) $\overrightarrow{XY} = \overrightarrow{XA} + \overrightarrow{AY}$ (Go from X to A, then from A to Y)

$\overrightarrow{OX} = \frac{3}{5}a$, so $\overrightarrow{XA} = \frac{2}{5}a$

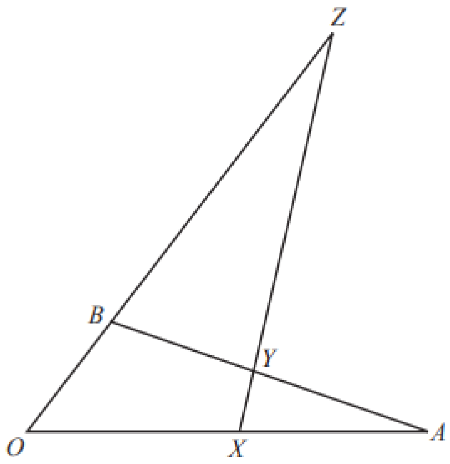
Since Y is the midpoint of AB, $\overrightarrow{AY} = \frac{1}{2}\overrightarrow{AB}$

$\overrightarrow{XY} = \frac{2}{5}a + \frac{1}{2}\overrightarrow{AB}$

$\overrightarrow{XY} = \frac{2}{5}a + \frac{1}{2}(b - a)$

$\overrightarrow{XY} = \frac{2}{5}a + \frac{1}{2}b - \frac{1}{2}a$

$\overrightarrow{XY} = \frac{1}{2}b - \frac{1}{10}a$



The lines OB and XY are extended to meet at the point Z. It is given that $\overrightarrow{YZ} = \lambda\overrightarrow{XY}$ and $\overrightarrow{BZ} = \mu\mathbf{b}$.

(c) Find $\overrightarrow{XZ}$ in terms of λ, $\mathbf{a}$ and $\mathbf{b}$. [2]

$\overrightarrow{XZ} = \overrightarrow{XY} + \overrightarrow{YZ}$ (Start at X, go to Y, then to Z)

We already know $\overrightarrow{XY}$ and $\overrightarrow{YZ} = \lambda\overrightarrow{XY}$ is given in question

$\overrightarrow{XZ} = \frac{1}{2}b - \frac{1}{10}a + \lambda(\frac{1}{2}b - \frac{1}{10}a)$

$\overrightarrow{XZ} = (\lambda + 1)(\frac{1}{2}b - \frac{1}{10}a)$

(d) Find $\overrightarrow{XZ}$ in terms of μ, $\mathbf{a}$ and $\mathbf{b}$. [2]

$\overrightarrow{XZ} = \overrightarrow{XO} + \overrightarrow{OB} + \overrightarrow{BZ}$ (Start at X, go to O, then to B, then to Z)

$\overrightarrow{OX} = \frac{3}{5}a$, so $\overrightarrow{XO} = -\frac{3}{5}a$, $\overrightarrow{OB} = b$, $\overrightarrow{BZ} = \mu b$

$\overrightarrow{XZ} = -\frac{3}{5}a + b + \mu b$

$\overrightarrow{XZ} = -\frac{3}{5}a + (1 + \mu)b$

(e) Hence find the values of λ and μ. [3]

We have 2 expressions for $\overrightarrow{XZ}$ so we can compare the 2 of them

$(\lambda + 1)(\frac{1}{2}b - \frac{1}{10}a) = -\frac{3}{5}a + (1 + \mu)b$

Let's compare a

$(\lambda + 1)(-\frac{1}{10}a) = -\frac{3}{5}a$

$-\frac{\lambda+1}{10} = -\frac{3}{5}$

$\lambda + 1 = 6$

$\lambda = 5$

Comparing b,

$(\lambda + 1)(\frac{1}{2}b) = (1 + \mu)b$

$(6)(\frac{1}{2}b) = (1 + \mu)b$

$3b = (1 + \mu)b$

$3 = 1 + \mu$

$\mu = 2$

9 In this question lengths are in centimetres and time is in seconds.

A particle P moves in a straight line such that its displacement s, from a fixed point at a time t, is given by $s = 3(t+2)(t-4)^2$ for $0 \leqslant t \leqslant 5$.

(a) Find the values of t for which the velocity, v, of P is zero. [4]

$s = (3t + 6)(t - 4)^2$

$v = \frac{ds}{dt} = 3(t - 4)^2 + (3t + 6)[2(t - 4) \times 1]$

$v = 3(t - 4)^2 + (6t + 12)(t - 4)$

Factor out $t - 4$

$v = (t - 4)[3(t - 4) + 6t + 12]$

$v = (t - 4)[3t - 12 + 6t + 12]$

$v = (t - 4)9t$

When $v = 0,\ (t - 4)9t = 0$

$9t = 0 \ or \ t - 4 = 0$

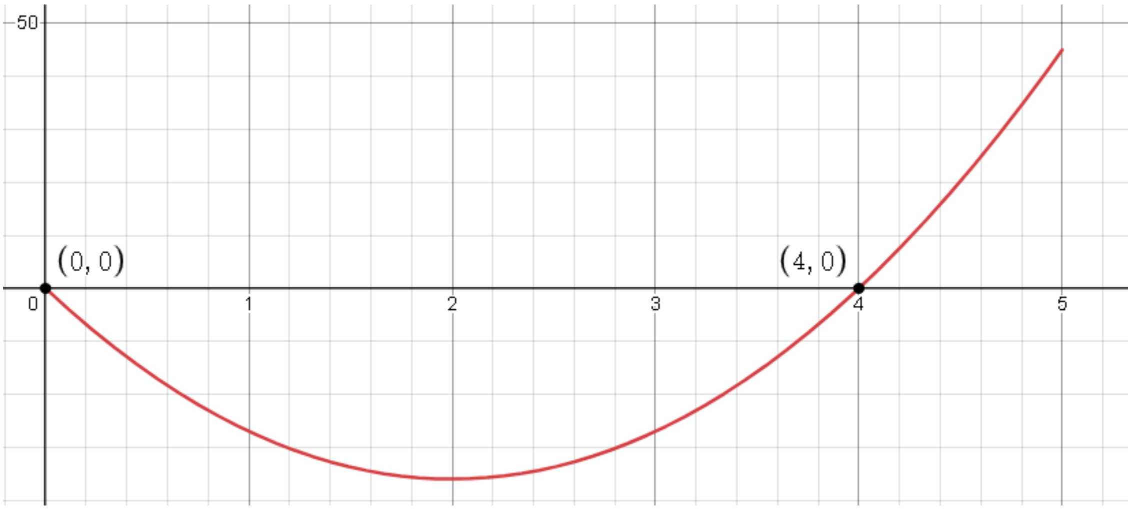
$t = 0 \ or \ t = 4$

(b) On the axes below, sketch the displacement–time graph of P, stating the intercepts with the axes. [3]

https://www.desmos.com/calculator/1lsjnga4br

Don’t forget your interception points!!

(c) On the axes below, sketch the velocity–time graph of P, stating the intercepts with the axes. [2]



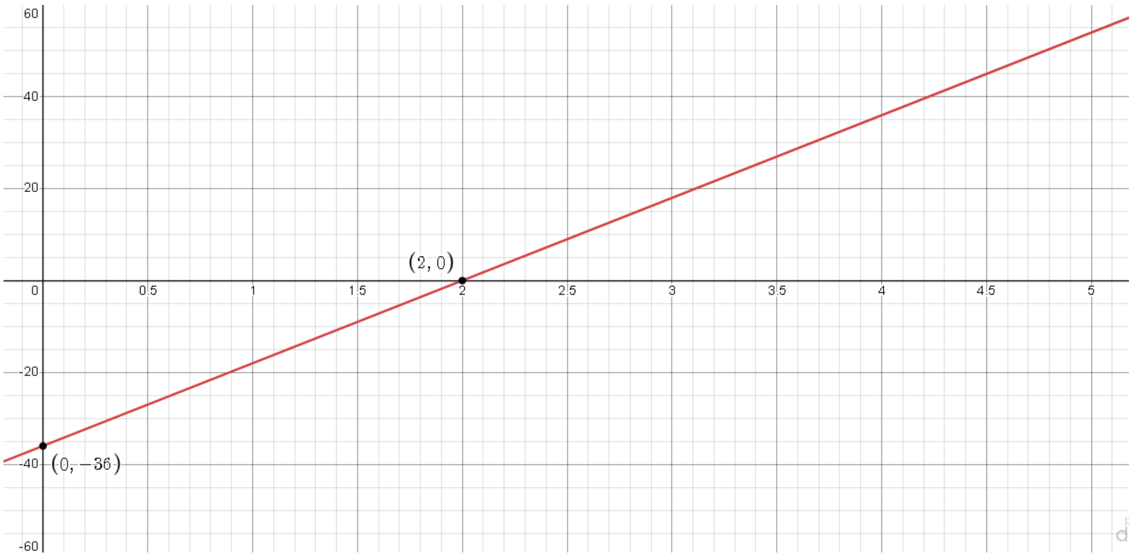
https://www.desmos.com/calculator/vnvaslekyz

(d) **(i)** Find an expression for the acceleration of P at time t. [1]

$$v = (t-4)9t = 9t^2 - 36t$$

$$a = \frac{dv}{dt} = 18t - 36$$

(ii) Hence, on the axes below, sketch the acceleration–time graph of P, stating the intercepts with the axes. [2]



https://www.desmos.com/calculator/gzw0k9cn0h

10 (a) Show that $\cos^4\theta - \sin^4\theta + 1 = 2\cos^2\theta$. [3]

$a^2 - b^2 = (a-b)(a+b)$

So, $\cos^4\theta - \sin^4\theta = (\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)$

Since $\cos^2\theta + \sin^2\theta = 1$,

$\cos^4\theta - \sin^4\theta = (\cos^2\theta - \sin^2\theta) \times 1 = \cos^2\theta - \sin^2\theta$

$\sin^2\theta = 1 - \cos^2\theta$

Now back to the original equation and substituting our new values,

$\cos^2\theta - (1 - \cos^2\theta) + 1$

$\cos^2\theta - 1 + \cos^2\theta + 1 = 2\cos^2\theta$

(b) Solve the equation $\cos^4\frac{\phi}{3} - \sin^4\frac{\phi}{3} + 1 = \frac{1}{2}$, for $-3\pi < \phi < 3\pi$, giving your answers in terms of π. [5]

First, don't forget to change your calculator to radians!!

Using part a,

$2\cos^2\frac{\phi}{3} = \frac{1}{2}$

$\cos^2\frac{\phi}{3} = \frac{1}{4}$

$\cos\frac{\phi}{3} = \pm\frac{1}{2}$

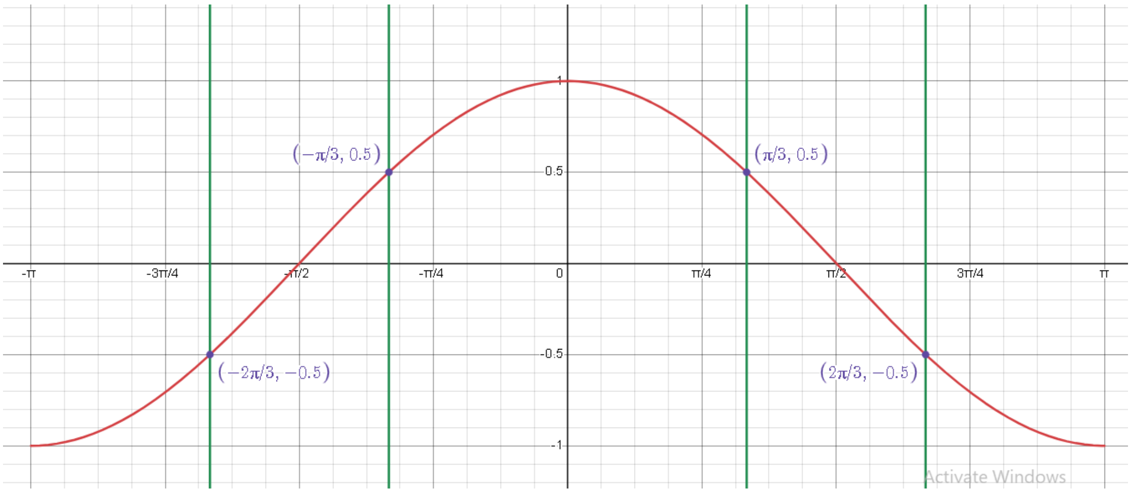
Let $x = \frac{\phi}{3}$, $\phi = 3x$

$-3\pi < \phi < 3\pi$

$-3\pi < 3x < 3\pi$

$-\pi < x < \pi$

Now sketching the graph of $\cos x = \pm\frac{1}{2}$ and using our calculator,



https://www.desmos.com/calculator/3ppkb6jfy7

We get 4 solutions of $x = -\frac{2\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}, and\ \frac{2\pi}{3}$

Substitute these values back into $\phi = 3x$ and we get

$\phi = -2\pi, -\pi, \pi, and\ 2\pi$

Additional notes

Websites and resources used:

- Desmos graphing calculator

If you find any errors or mistakes within this paper, please contact us and we will fix them as soon as possible.